Exercise 29

For the following exercises, solve the equations over the complex numbers.

$$x^2 + 27 = 0$$

Solution

Isolate the term with the variable by subtracting 27 from both sides.

$$x^2 = -27$$

Take the square root of both sides.

$$\sqrt{x^2} = \sqrt{-27}$$

$$= \sqrt{9(-1)(3)}$$

$$= \sqrt{9}\sqrt{-1}\sqrt{3}$$

$$= 3i\sqrt{3}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around x.

$$|x| = 3i\sqrt{3}$$

Remove the absolute value sign by placing \pm on the right side.

$$x = \pm 3i\sqrt{3}$$

Therefore, $x = \{-3i\sqrt{3}, 3i\sqrt{3}\}.$